Que: $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, show that $\frac{x^2+y^2}{x^2-y^2} = \frac{7\sqrt{3}}{12}$

Soln: L.H.S $= \frac{x^2+y^2}{x^2-y^2}$

$$= \frac{\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 + \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2}{\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 - \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2}$$

$$= \frac{\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 - 2\,\frac{\cancel{(\sqrt{3}+1)}}{\cancel{(\sqrt{3}-1)}} \times \frac{\cancel{(\sqrt{3}-1)}}{\cancel{\sqrt{3}+1}}}{\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}$$

$$= \frac{\left\{\frac{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}\right\}^2 - 2}{\left\{\frac{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}\right\}\left\{\frac{(\sqrt{3}+1)^2-(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}\right\}}$$

$$= \frac{\left\{\frac{(\sqrt{3})^2+\cancel{2\sqrt{3}}+1+(\sqrt{3})^2-\cancel{2\sqrt{3}}+1}{(\sqrt{3})^2-1}\right\}^2 - 2}{\frac{(\sqrt{3})^2+\cancel{2\sqrt{3}}+1+(\sqrt{3})^2-\cancel{2\sqrt{3}}+1}{(\sqrt{3})^2-(1)^2} \times \frac{\cancel{(\sqrt{3})^2}+2\sqrt{3}+\cancel{1}-\cancel{(\sqrt{3})^2}+2\sqrt{3}-\cancel{1}}{(\sqrt{3})^2-(1)^2}}$$

$$= \frac{\left\{\frac{8}{3-1}\right\}^2 - 2}{\frac{8}{3-1} \times \frac{4\sqrt{3}}{3-1}}$$

$$= \frac{\left(\frac{\cancel{8}^{\,4}}{\cancel{2}}\right)^2 - 2}{\frac{\cancel{8}^{\,4}}{\cancel{2}} \times \frac{\cancel{4}^{\,2}\sqrt{3}}{\cancel{2}}}$$

$$= \frac{16-2}{8\sqrt{3}} = \frac{14}{8\sqrt{3}} = \frac{7\sqrt{3}}{12}$$

$\cancel{= \frac{14\sqrt{3}}{24}}$

L.H.S (proved)